CS 2800 – Fall 2010
Review Problems for the Final

1. Convert the following statements into propositional logic and determine if the given argument is sound:

   Rainy days make gardens grow.
   Gardens don't grow if it is not hot.
   It always rains on a day that is not hot.
   Therefore, if it is not hot, then it is hot.

2. We define the set $S = \{x | x \notin S\}$. Does $S \in S$? Does $S \notin S$?

3. Show that if A and B are countable sets, then $A \times B$ is also countable.

4. Show that the power set of the set of positive integers $\mathbb{Z}^+$ is uncountable.

5. What is the best big-O function for

   (a) $n^3 + \sin n^7$

   (b) $(x + 2) \log_2 (x^2 + 1) + \log_2 (x^3 + 1)$

6. What is the worst-case time complexity of the following algorithms?

   (a) One that prints out all the ways to place the numbers $1, 2, \ldots, n$ in a row.

   (b) One that enumerates all the subsets of a given finite set of size n.

7. Prove that if n is an integer that is not a multiple of 3, then $n^2 \equiv 1 \pmod 3$.

8. Explain in words the difference between $a|b$ and $\frac{b}{a}$.

9. Find an inverse of 17 modulo 19.

10. Show using induction that $11|(10^{2k+1} + 1)$ for all non-negative integers k.

11. Show using induction that for all positive integers n:

$$1 - 2 + 2^2 - 2^3 + \ldots + (-1)^n 2^n = \frac{2^{n+1}(-1)^n + 1}{3}$$

12. How many strings of eight English letters are there

    (a) if letters can be repeated?

    (b) if no letter can be repeated?

    (c) that start with X, if letters can be repeated?

    (d) that start with X, if no letter can be repeated?

    (e) that start and end with X, if letters can be repeated?

    (f) that start with the letters BO (in that order), if letters can be repeated?

    (g) that start and end with the letters BO (in that order), if letters can be repeated?

    (h) that start or end with the letters BO (in that order), if letters can be repeated?

13. Show that there are at least 6 people in California (population 36 million) with the same three initials who were born on the same day of the year (but not necessarily the same year). Assume everyone has three initials.

14. How many different strings can be made from the letters in *ORONO* using some or all the letters?

15. What is the probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

16. In the following question, an experiment consists of picking a bit string of length 5 at random. Consider the following events:

    - E_1: the bit string chosen begins with a 1.
    - E_2: the bit string chosen ends with a 1.
    - E_3: the bit string chosen has exactly three 1s.

    (a) Find $p(E_1)$, $p(E_2)$ and $p(E_3)$.

    (b) Find $p(E_1|E_3)$.

    (c) Find $p(E_2|E_1 \cap E_3)$.

    (d) Are E_1 and E_2 independent? Justify your answer.

    (e) Are E_2 and E_3 independent? Justify your answer.

17. n people check in their coats in an opera house. Unfortunately, there is a mix-up in the coat room and after the show, everyone receives a coat chosen uniformly at random. On average, how many people can expect to get their own coat? Give a rigorous derivation of your answer.

18. Define $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$. What is the language L generated by the grammar $G = (V, T, S, P)$ when P, the set of productions consists of:

    (a) $S \to AB, A \to ab, B \to bb$

    (b) $S \to AB, S \to aA, A \to a, B \to ba$